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A parity conserving dimer model with infinitely many absorbing states

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Abstract. We propose and study a model where two aspects are present: parity conservation and infinitely many absorbing states. Whereas steady-state simulations show that the static critical behaviour is not affected by the presence of multiple absorbing configurations, the influence of the initial state associated with the presence of slowly decaying memory effects is clearly displayed in time dependent simulations. We report results of a detailed investigation of the dependence of critical spreading exponents on the initial particle density.

PACS. 64.60.-i General studies of phase transitions – 64.60.Ht Dynamic critical phenomena – 02.50.-r Probability theory, stochastic processes, and statistics

Various nonequilibrium models exhibiting phase transitions from an active to an absorbing phase have been extensively investigated over recent years [1–4].

Most of them [5–10] have been shown to belong to the universality class of directed percolation (DP) [11]. These studies gave support to the conjecture made by Janssen and Grassberger [12,13] that continuous transitions to an unique absorbing state generically fall in the DP class. The same applies, as far as the static critical behaviour is concerned, to various models with multiple absorbing states [14–16]. However, the dynamic critical properties of these systems are found to be non-universal: in fact, spreading critical exponents depend on the initial density and obey a generalized hyperscaling relation derived by Mendes *et al.* [16]. Some systems, first claimed to have a different critical behaviour, were later included in the DP class, upon a more careful analysis [8,9].

For sometime the only accepted exceptions to the DP universality class were the models A and B introduced by Grassberger and collaborators [17,18]: they both have a doubly degenerated absorbing state and modulo 2 parity conservation of kinks (-00- and -11-s). Since then, a growing number of models whose critical behaviour falls in the parity conserving (PC) universality class have been studied. One of the first to be considered was the branching annihilating random walk with an even number m of offsprings (BAW) [10,19–21]: in this case, the parity of the particles is conserved and there is a single absorbing state. Another system extensively studied is the nonequilibrium Ising model with combined spin-flip and spin-exchange dynamics (NEKIM) [22]; here, the absorbing state is symmetrically doubly degenerated and the kinks have local parity conservation, similarly to what happens with the A and B models referred above. Other models have since been discovered and shown to display PC critical behaviour: the interacting monomer-dimer model [23] and generalizations of the Domany-Kinzel automaton with n equivalent absorbing states [24]. In these models, parity is not strictly conserved but there is complete symmetry among absorbing states; actually, if this symmetry is broken, DP critical behaviour is recovered.

Therefore, the question about the essential feature behind PC critical behaviour is not yet settled. Whereas in BAW models mass conservation of modulo 2 is determinant, in the other cases symmetry among absorbing states plays the relevant role. In the NEKIM model, parity conservation in the number of kinks is not sufficient to ensure PC-like behaviour, if such symmetry is broken[22].

Recently, Inui and Tretyakov [25] presented a new version of a contact process with parity conservation, which displays a phase transition for $m = 2$, contrarily to what happens with the original version of BAW. They claim, on the basis of Padé approximants and also numerical simulations, that the order parameter exponent β should be 1, while the values estimated by different authors are consistently smaller [19,23,24].

In this article, we consider a model that resembles the pair-contact process (PCP) introduced by Jensen [14], but where the number of dimers is conserved modulo 2. Dimers cannot be generated spontaneously, therefore any configuration with isolated particles and empty sites is absorbing, and there is an infinite number of such configurations [26]. This enables one to study the way initial configurations

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affect the spreading critical exponents in a system that belongs to the PC-universality class.

Initial conditions were shown [27] to affect time-dependent critical exponents that describe the damage spreading transition (DS) in cases where this one coincides with the ordinary critical point; in DS, one follows the evolution of two replicas, so this can be seen as a multicomponent system, and any state where both replicas are identical is considered as absorbing.

We start by briefly reviewing the dynamic rules for the PCP. This is a lattice model which can be described by a 2-state variable $\sigma_i = 0, 1$. Nearest-neighbour pairs of particles (dimers) annihilate each other with probability p or create,with probability $1-p$, a particle at one of the adjacent (vacant) sites to the dimer. The annihilation of a pair may imply the loss of one or two other pairs, if one (or two) of the nearest-neighbour sites to the chosen dimer happen to be occupied; analogously when a particle is created adjacent to a dimer, the number of pairs increases by one or two, depending on the occupancy of the other nearest-neighbour to the site that is being occupied. It is therefore clear that there is no parity conservation in the number of dimers and the model displays a DP-like phase transition from an active state with a nonzero concentration of dimers (for $p < p_c$) to a phase with infinitely many absorbing states (for $p > p_c$). Critical spreading exponents are non-universal depending upon the nature of the initial configuration.

In the one-dimensional model we now introduce, a phase with infinitely many absorbing states is also present. On the other hand, dynamic rules are such that there is parity conservation in the number of dimers. This seems to be the most relevant feature; indeed, we find that the static critical behaviour of this model is PC-like, thus it is not affected by the presence of multiple absorbing states.

The basic processes are again annihilation and creation, which are attempted with probabilities p and $1-p$, respectively. If one chooses to annihilate, then one dimer (represented by $\bullet = \bullet$) is selected at random and one looks for an adjacent dimer. If the pair of dimers is surrounded by empty sites ($\circ$), then the two dimers are annihilated and the respective sites become empty ($- \circ - \bullet = \bullet = \bullet - \circ - \rightarrow - \circ - \circ - \circ - \circ - \circ -$); otherwise, annihilation is produced by simply vacating the site that is shared by the two dimers, leaving the other sites unchanged ($- \circ - \bullet = \bullet = \bullet = \bullet - \rightarrow - \circ - \bullet - \circ - \bullet = \bullet -$). In this way, the number of dimers is conserved modulo 2. In case of a creation attempt, then if the nearest and next-nearest-neighbour sites of a selected dimer are, respectively, vacant and occupied, a pair of dimers is produced by simply occupying that vacant site ($-\bullet = \bullet - \circ - \bullet - \circ - \rightarrow - \bullet = \bullet = \bullet = \bullet - \circ -$); another possibility requires the presence of three vacant sites adjacent to a dimer, in which case a pair of dimers is created (with probability α) by filling the nearest and next-nearest-neighbour sites and leaving the third vacant site unchanged ($-\bullet = \bullet - \circ - \circ - \circ - \rightarrow - \bullet = \bullet = \bullet = \bullet - \circ -$). When $\alpha < 1$, growth is less effective if vacant sites (rather than isolated particles) exist in the environment that surrounds dimers, whereas in the case $\alpha = 1$ growth is not affected by the concentration of isolated particles in the environment. In the present work, we have fixed $\alpha = 1/2$.

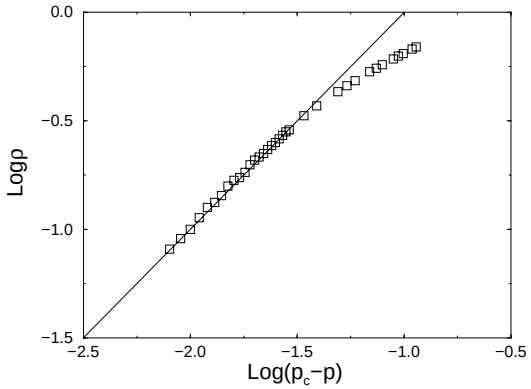

Fig. 1. Log_{10}-log_{10} plot of the concentration of dimers, ρ *versus* $p_c - p$, with $p_c = 0.2990$.

There is in fact a conservation law in the dynamics above [28]. This can be seen in the following way: divide the chain into two sublattices A and B and let N_{AB} (N_{BA}) be the total number of dimers with left particle on the A (B) sublattice. Then $N = N_{AB} - N_{BA}$ is conserved since dimers are created and destroyed in pairs on adjacent pairs of sites. Thus configurations with different values of N are not connected by the dynamics.

With these rules, diffusion is only indirectly present. Also, the annihilation of isolated dimers is here possible only through successive processes of creation and annihilation, which slows down the dynamical processes leading to absorbing configurations. One can of course enlarge the parameter space to include diffusion or longer range processes and consider a dynamics that mixes up sectors in the phase space with different values of N. We have not done it in the present work. In the absence of such processes, creation of dimers is inhibited in configurations where a pair of vacant sites is surrounded by occupied sites. Initial configurations have therefore been chosen such that this situation does not occur in the dynamical process, this meaning a restriction to the $N = 0$ sector.

The order parameter of the system is the concentration of dimers, which vanishes algebraically as p approaches the critical probability p_c:

$$\rho \sim (p_c - p)^{\beta} \tag{1}$$

where β is the order parameter exponent.

Simulations were started with a fully occupied lattice and the concentration averaged over a long period of time once the steady state has been reached.

In Figure 1 we show a log-log plot of the steady state concentration as a function of $p_c - p$ for system size $L = 2000$; time varied from $t = 5000$ to $t = 2 \times 10^5$ MCS closest to $p_c (= 0.2990(10))$ and we averaged over around 1000 independent samples which had not yet entered the absorbing state. From the slope of the data we estimate $\beta = 0.98(5)$, a value which agrees with the results of [25]

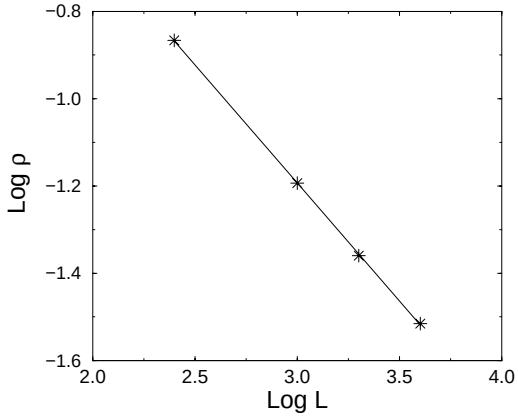

Fig. 2. Log_{10}-$\log_{10}$ plot of $\rho(p_c, L)$ *versus* L (L = 250, 500, 1000 and 2000).

and is slightly above the values obtained by other authors for models in the PC universality class. Determining the critical point by steady state simulations requires an accurate measurement of the order parameter near criticality, which becomes rather difficult due to the critical slowing down.

We have complemented this study with a finite-size scaling analysis based on the ansatz that the order parameter depends on system size L through the ratio of L and the correlation length $\xi \sim \Delta^{-\nu_\perp}$:

$$\rho(p, L) \sim L^{-\beta/\nu_\perp} f(\Delta L^{1/\nu_\perp}) \tag{2}$$

(with $f(x) \propto x^\beta$ for $x \to \infty$, so that (1) is recovered when $L \to \infty$).

Accordingly,

$$\rho(p_c, L) \propto L^{-\beta/\nu_\perp}. \tag{3}$$

In Figure 2, we show a log-log plot of $\rho(p_c, L)$ as a function of L; a straight line is obtained, from the slope of which we estimate $\beta/\nu_\perp = 0.54$ (4), to be compared with 0.48 and 0.50 as obtained in [23,19], respectively.

We also report results of critical spreading, *i.e.* the evolution of the critical system from a configuration that is very close to an absorbing state. In the present model, there are many absorbing configurations and some critical spreading exponents are indeed dependent upon the density of particles in the initial state. The quantities that are usually considered in these studies are the surviving probability $P(t)$, the number of dimers $n(t)$ averaged over all runs, and the mean-square distance of spreading $R^2(t)$ averaged over the surviving runs. At criticality, these quantities obey, in the long time limit, $P(t) \sim t^{-\delta}$, $n(t) \sim t^\eta$ and $R^2(t) \sim t^z$, and the corresponding exponents can be obtained from the straight lines shown in double-logarithmic plots of the quantities against time. More precise estimates are usually obtained by looking at local slopes, for example $-\delta(t) = \ln[P(t)/P(t/m)]/\ln(m)$. In a plot of the local slopes *vs.* $1/t$, the critical exponents are given by the intercept of the curves for p_c with the vertical axis, whereas curves for $p > p_c$ ($p < p_c$) veer downward (upward). This often constitutes a rather accurate method for the determination of p_c. It is certainly the case when one or just a few absorbing states are present [23,29]. Also in some systems with infinitely many absorbing states, the critical point was shown not to depend on the initial configuration and can therefore be located by the procedure we just described [16]. In other cases, indications of a slight dependence of the critical point (as determined by the above time-dependent analysis) on the initial configuration have been found [30,31] and can be attributed to slowly decaying memory effects [32]. This seems to occur also in the present case.

In these simulations we started the system in a way that a sublattice of alternating sites is vacant except for one central site, and the sites of the other sublattice are occupied with probability q; the two nearest-neighbours of the central site are also occupied, thus constituting a perturbation of exactly two dimers. The size of the lattice was chosen such that the spreading never hits the boundaries. For different q values, a number of independent runs, typically 10^7, were performed, up to 4000 time steps each, and various values of p.

A local slope analysis for $q = 0.43$ leads to the estimate $p_c = 0.2995(5)$, which is fully consistent with the p_c estimate from steady-state simulations, above; the estimates for the dynamic scaling exponents are $\eta' = 0.005(5)$, $\delta' = 0.291(6)$, $z'/2 = 0.570(6)$ and agree well with previous results for models in the PC universality class [19] ($\eta = 0.000(1)$, $\delta = 0.285(2)$, $z/2 = 0.571(1)$). This is similar to what happens in the pair-contact process: DP time-dependent exponents are found if one studies spreading of a perturbation to the "natural" configuration, the one spontaneously generated by the critical dynamics. The estimate $q^{\mathrm{nat}} = 0.43(2)$ (q^{nat} meaning the value of q that corresponds to the density in the natural configuration) is in good agreement, within numerical accuracy, with the one obtained by generating the natural configuration in samples of size $L = 2000$, starting with a full lattice and using periodic boundary conditions.

For different q values, power laws still hold, but with different critical spreading exponents, and a slight shift of the critical point. In Figure 3 we show the local slope analysis for $q = 0.2$, leading to the following estimates: $\delta' = 0.366(6)$, $\eta' = -0.069(5)$, $z'/2 = 0.567(6)$, $p'_c = 0.3000(5)$. The generalized hyperscaling relation $\eta' + \delta' - z'/2 = -\delta$ [16] is well satisfied by these values.

Our results for the q-dependence of p'_c, δ', η' and $z'/2$ are given in Table 1. A check of the generalized hyperscaling is also included. Indeed, the exponent z' does not present a significative dependence on the parameter q, given the numerical errors. Then, as expected, the exponent governing the population growth in surviving critical trials, $\delta' + \eta'$, does not depend on the initial particle concentration.

In conclusion, we have performed a similar study to what has been done before [14] for the PCP (and other models with multiple absorbing states [16]) but now a system with parity conservation (in the number of dimers) is

Table 1. The table shows the q-dependence of the critical parameter p_c' and critical exponents δ', η' and $z'/2$. A test of generalized hyperscaling is also included.

q	p_c'	δ'	η'	$z'/2$	$\eta'+\delta'-z'/2+\delta$
DP	-	0.1597(3)	0.312(2)	0.632(1)	0.000(1)
PC	-	0.285(2)	0.000(1)	0.571(1)	−0.001(1)
0.1	0.3005(5)	0.400(6)	−0.106(5)	0.570(6)	0.01(2)
0.2	0.3000(5)	0.366(6)	−0.069(5)	0.567(6)	0.02(2)
0.43	0.2995(5)	0.291(6)	0.005(5)	0.570(6)	0.01(2)
0.45	0.29935(5)	0.284(6)	0.014(5)	0.570(6)	0.01(2)
0.5	0.2990(5)	0.271(6)	0.033(5)	0.572(6)	0.02(2)
0.6	0.2990(5)	0.239(6)	0.062(5)	0.573(6)	0.01(2)
0.8	0.2985(5)	0.181(6)	0.126(5)	0.576(6)	0.02(2)
0.9	0.2985(5)	0.156(6)	0.154(5)	0.579(6)	0.02(2)

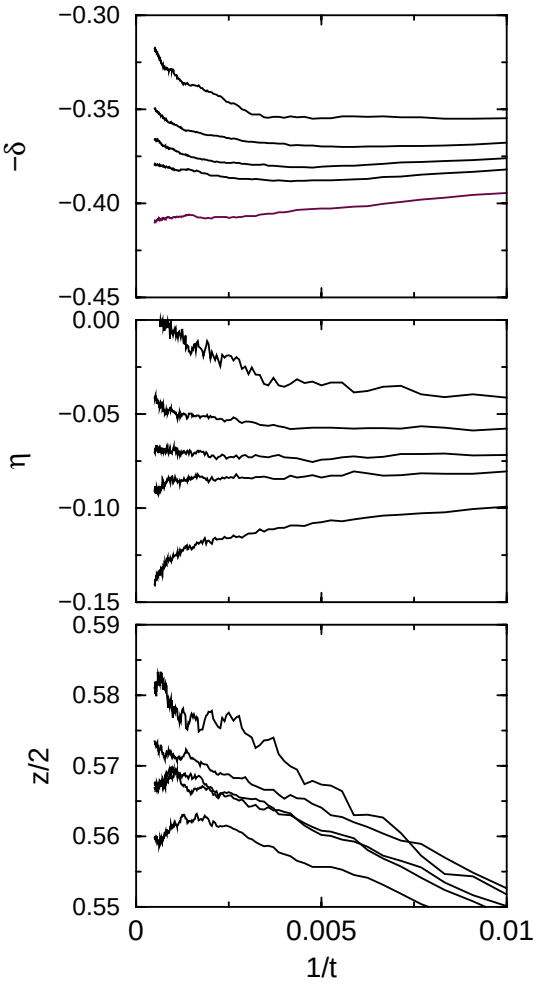

Fig. 3. Local slopes $-\delta'(t)$ (upper panel), $\eta'(t)$ (midle-panel) and $z'/2(t)$ (lower panel) for $q = 0.2$. Each panel contains five curves with, from bottom to top, $p = 0.2980$, 0.2990, 0.3000, 0.3010 and 0.3020.

for the first time investigated. A field theory formalism to appropriately describe this situation is now required.

Whereas the presence of multiple absorbing states does not affect the static behaviour, spreading critical behaviour is expressed by power laws whose exponents depend on the initial particle density ($= q/2$). There is also a monotonic shift in p_c', which, when found in other systems, has been attributed [31] to slowly decaying memory effects displayed by the non-order field. The generalized hyperscaling relation is verified.

The role played by the conservation law in N is not clear from the present study [33]. Further investigation by considering a dynamics that mixes up sectors with different N values is planned.

A possible mapping of this system to a SOC model[34] is currently under study.

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